PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

**Semester One Examination, 2018**

**Question/Answer booklet**

# MATHEMATICS SPECIALIST UNIT 1

## Section Two: Calculator-assumed

Your Name

Your Teacher's Name

## Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***
This Question/Answer booklet
Formula sheet (retained from Section One)

***To be provided by the candidate***
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

| Question | Marks | Question | Marks |
|---|---|---|---|
| 8 | | 15 | |
| 9 | | 16 | |
| 10 | | 17 | |
| 11 | | 18 | |
| 12 | | 19 | |
| 13 | | 20 | |
| 14 | | | |

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 7 | 7 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 94 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

**Section Two: Calculator-assumed** **(94 Marks)**

This section has **Thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes

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**Question 8** **(6 marks)**

Determine whether each of the following statement is true or false. Prove in general if the statement is correct. Use counter examples if the statement is false.

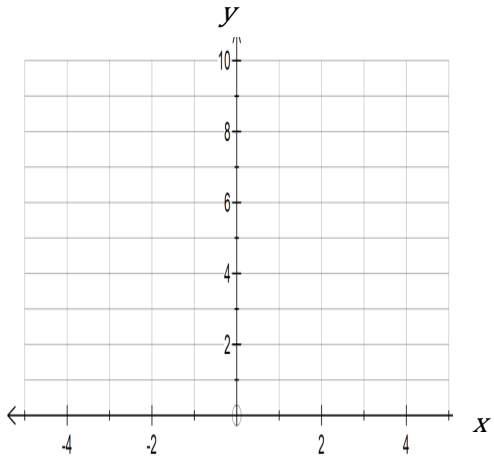
(a) A quadrilateral with a right angle must be a rectangle.

(b) For all positive numbers $a$, $a^2 \leq a^3$

(c) $x^2 + x + 11$ always gives a prime number.

**Question 9** **(8 marks)**

(a) The position vectors of points $A$, $B$, and $C$ are $\boldsymbol{i}$, $-\boldsymbol{i} + 3\boldsymbol{j}$ and $-5\boldsymbol{i} + 9\boldsymbol{j}$ respectively. Show that the points $A, B$, and $C$ are collinear and hence state the ratio of $AB : BC$. (4 marks)




(b) Using the information in part (a). It is given that $OBCD$ is a parallelogram and that $E$ is the point such that $\overrightarrow{DB} = \frac{1}{3}\overrightarrow{ED}$. Find the position vectors of $D$ and $E$. (4 marks)

**Question 10** **(12 marks)**

(a) The velocity of a boat $v_1$ changes from 25 knots at a bearing of 38° to $v_2$ which is 15 knots at a bearing of 107°. What is the change in the velocity? Draw a diagram for this situation. State the changes in the speed and the direction. [Hint: find $v_2 - v_1$ .] (5 marks)

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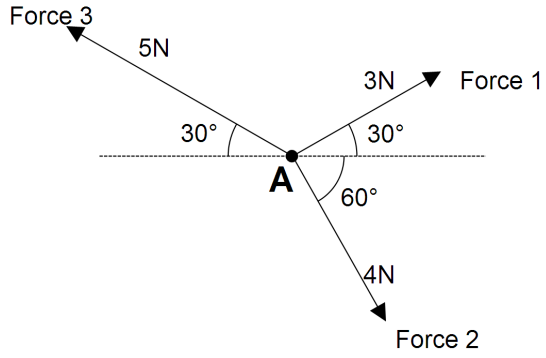
(b) Three forces act on the point *A* as shown. What is the magnitude and direction of the resultant force acting on *A*? (7 marks)

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**Question 11** **(8 marks)**

An aircraft is to be flown from Perth (P) to a bush fire (F) that is 180 km away on bearing of $160^{o}$. If a wind of 30km/hr is blowing **<u>from</u>** a bearing of $50^{o}$ and the aircraft can maintain a steady speed of 250km/hr in still air find:

(a) The bearing on which the plane must be set (Draw a diagram) (4 marks)

(b) The actual speed of the plane (2 marks)

(c) The time of the journey (2 marks)

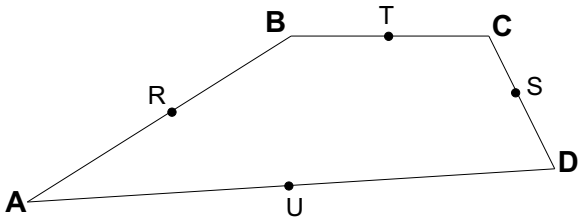
**See next page**

**Question 12** **(10 marks)**

(a) Find the values of $\lambda$ and $\mu$ given that $\boldsymbol{a}$ and $\boldsymbol{b}$ are non-parallel vectors and (4 marks)

$$2\lambda\boldsymbol{a} + 3\mu\boldsymbol{a} - \mu\boldsymbol{b} + 2\boldsymbol{b} = \lambda\boldsymbol{b} + 2\boldsymbol{a}$$

(b) In the quadrilateral $ABCD$ shown below $\overrightarrow{AB} = b$, $\overrightarrow{AC} = c$, $\overrightarrow{AD} = d$. The points $R, T, S$ and $U$ are the midpoints of the sides shown in the diagram. Show, using vectors $\boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{d}$ that $\overrightarrow{TS}$ is parallel to $\overrightarrow{RU}$. (6 marks)



**Question 13** **(8 marks)**

(a) For digits {1, 2, 3, 4, 5, 6}

(i) How many numbers can be formed in total, if each digit is only used at most once?
[Hint: consider all 1 to 6-digit numbers]. (2 marks)

(ii) How many of the above are less than 600,000? (2 marks)

(b) For digits {1, 2, 3, 4, 5, 5, 6, 6}, how many distinct four-digit numbers can be formed?

(4 marks)

**See next page**

**Question 14** **(5 marks)**

A set S contains all the integers between 3 and 122 inclusive.

[5 marks = 1, 2, 2]

(a) how many numbers in set S are multiples of 6? (1 mark)

[1 mark]

(b) how many numbers in set S are multiples of 6 or 8? (2 marks)

[2 marks]

(c) how many numbers in set S are multiples of either 6 or 8 but not both? (2 marks)

**Question 15** **(3 marks)**

Show that if 50 different integers are selected from the set {1, 2, 3, ..., 97, 98}, there will be at least two integers whose sum is 99.

**See next page**

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**Question 16** **(6 marks)**

(a) Determine the angle between the vectors $< -12, 7 >$ and $< 3, 8 >$. (2 marks)

(b) Determine the value of $a$ so that the vectors $< 7, a >$ and $< 10, 4 >$ are perpendicular. (2 marks)

(c) Determine the exact scalar projection of $< 3, -5 >$ on $< -8, 4 >$. (2 marks)

**See next page**

**Question 17** **(10 marks)**

How many ways can the letters of the word ***TRIANGLE*** be arranged

(a) if there is no restriction? (1 mark)

(b) if the first three letters must be ***RAN*** (in that exact order)? (2 marks)

(c) if the first three letters must be ***RAN*** (in any order) and the next three letters must be ***TRI*** (in any order)? (2 marks)

(d) if the order and the placement of the vowels ***IAE*** cannot be changed? (i.e. ***IAE*** stay where they are as in *TR**IA**NGL**E***) (2 marks)

(e) if the order of the vowels ***IAE*** cannot be changed, though their placement may (***IAE***TRNGL and TR***IA***NG***E***L are acceptable but ***EIA***TRNGL and TR***IE***NGL***A*** are not)? (3 marks)

**See next page**

**Question 18** **(6 marks)**

Airports A and B are such that $\overrightarrow{AB} = 250i - 750j$ km. An aircraft is to be flown directly from A to B. In still air the aircraft can maintain a steady speed of 400km/hr. There is a wind blowing with velocity $13i + 9j$ km/hr.

(1) Find, in the form $ai + bj$, the velocity vector the pilot should set so that this velocity, together with the wind, causes the plane to travel directly from A to B. (4 marks)

(2) If the wind remains unchanged, in the form $ai + bj$, the velocity vector the pilot should now set to return directly from B to A. (2 marks)

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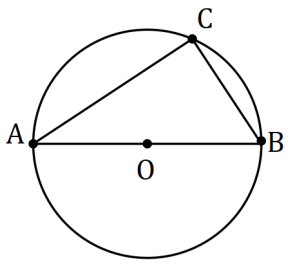
**Question 19** **(4 marks)**

Use a vector method to prove that the angle in a semi-circle is a right-angle.




Hint: Let $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OB} = \mathbf{b}$.

**See next page**

**Question 20** **(8 marks)**

(a) Show that $\frac{1}{(k+2)!} - \frac{k+1}{(k+3)!} = \frac{2}{(k+3)!}$ (2 marks)

(b) Prove by induction that, for all positive integers $n$

$$\frac{1\times 2^1}{(1+2)!} + \frac{2\times 2^2}{(2+2)!} + \frac{3\times 2^3}{(3+2)!} + \cdots + \frac{n\times 2^n}{(n+2)!} = 1 - \frac{2^{n+1}}{(n+2)!}$$ (6 marks)

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**End of Questions**

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Question number: _____

**Additional working space**

Question number: _____

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Question number: _____

## Acknowledgements



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